11. Generalized Exchange and Laws of Conservation

We will consider kinematic exchange between a system and the environment on the *Z*-level of rest-motion (Fig. 2.15).

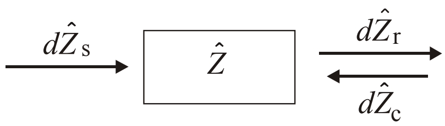

Fig. 2.15. A graph of *Z-level* exchange.

Let motion-rest $d\hat{Z}_s$ be transferred from the environment to the system and the amount $d\hat{Z}_r$ of motion-rest be transferred from the system to the environment along the kinetic channel and $d\hat{Z}_c$ be transferred by the system over the potential. If $\hat{Z} = \hat{P}$, then

$$d\hat{Z}_s = d\hat{P}_s, \quad d\hat{Z}_r = -rd\hat{\psi}, \quad \hat{Z}_r = -kd\hat{\Phi}, \tag{2.262}$$

where $\hat{P}$ *is a* parameter of any level of motion, *r* is kinetic resistance or kinetic elasticity, *k* is potential resistance or potential elasticity, $d\hat{\psi}$ and $d\hat{\Phi}$ are differentials of particular states.

In a general case, the resistances of the exchange channels depend on the state of the system, environment, and the character of the exchange channels; in the linear approximation they are constant. Their inverse values, *g* and C, will be called kinetic and potential conductivities, respectively.

Each of the differentials of exchange over a direct and two inverse channels determines the amount of mutual exchange equal to the difference of partial components of exchange. The rest-motion $md\hat{\upsilon}$ gained by the system is equal to the sum of exchanges in the three channels. Thus, we have

$$md\ddot{\upsilon} = dP_s + (-rd\ddot{\psi}) + (-kd\Phi). \tag{2.263}$$

Hence, we arrive at the equation of exchange in the form:

$$\frac{md\hat{\upsilon}}{dt} + \frac{rd\hat{\psi}}{dt} + \frac{kd\hat{\Phi}}{dt} = \frac{d\hat{P}_s}{dt} \tag{2.264}$$

or

$$\frac{md\hat{\upsilon}}{dt} + r\hat{\upsilon} + \frac{1}{C}\hat{\psi} = \hat{F}_s \tag{2.264a}$$

or

$$\frac{md^2\hat{\psi}}{dt^2} + r\frac{d\hat{\psi}}{dt} + \frac{1}{C}\hat{\psi} = \hat{F}_s. \tag{2.264b}$$

The equation of exchange is simultaneously the equation of the state of the system.

We will write the exchange-state equations for $\hat{S}$-, $\hat{P}$-, $\hat{F}$- and $\hat{D}$- levels:

$$\frac{md^2\hat{O}}{dt^2}+r\frac{d\hat{O}}{dt}+\frac{1}{C}\hat{O}=\hat{S}_s, \quad \frac{md^2\hat{\Phi}}{dt^2}+r\frac{d\hat{\Phi}}{dt}+\frac{1}{C}\hat{\Phi}=\hat{P}_s, \tag{2.265}$$

$$\frac{md^2\hat{\psi}}{dt^2}+r\frac{d\hat{\psi}}{dt}+\frac{1}{C}\hat{\psi}=\hat{F}_s, \quad \frac{md^2\hat{\upsilon}}{dt^2}+r\frac{d\hat{\upsilon}}{dt}+\frac{1}{C}\hat{\nu}=\hat{D}_s \tag{2.266}$$

or

$$m\hat{\psi}+r\hat{\Phi}+\frac{1}{C}\hat{O}=\hat{S}_s, \quad m\hat{\upsilon}+r\hat{\psi}+\frac{1}{C}\hat{\Phi}=\hat{P}_s, \tag{2.267}$$

$$m\hat{w}+r\hat{\upsilon}+\frac{1}{C}\hat{w}=\hat{F}_s, \quad m\hat{z}+r\hat{w}+\frac{1}{C}\hat{\upsilon}=\hat{D}_s. \tag{2.268}$$

In a broad sense, the first terms in the left-hand sides of the equations are kinetic momenta, the second and third terms are kinetic and potential momenta of the feedback with the environment.

If we introduce the generalized charge

$$\hat{Q}_m=\frac{m\hat{\upsilon}}{a}, \quad \hat{Q}_r=\frac{r\hat{\psi}}{a}, \quad \hat{Q}_c=\frac{1}{Ca}\hat{\Phi}, \tag{2.269}$$

where a is the characteristic length, then in terms of charges the equation for the $\hat{P}$ -level becomes:

$$\hat{Q}_s=\hat{Q}_m+\hat{Q}_r+\hat{Q}_c. \tag{2.270}$$

For the $\hat{F}$ -level it will be represented by the equation of current:

$$\hat{I}_s=\hat{I}_m+\hat{I}_r+\hat{I}_c. \tag{2.271}$$

Finally, on the $\hat{D}$ -level the equation takes the form:

$$\frac{d\hat{I}_s}{dt}=\frac{d\hat{I}_m}{dt}+\frac{d\hat{I}_r}{dt}+\frac{d\hat{I}_c}{dt}. \tag{2.272}$$

If the system is closed over the channel $\hat{D}_s$ ($\hat{D}_s=0$), it is closed over all overlying channels and in a general case, it is not closed over all underlying channels

Energy description of the levels $\hat{S}$, $\hat{P}$, $\hat{F}$ and $\hat{D}$ is expressed by

$$\hat{E}_s=\int\hat{S}_s d\hat{O}=\frac{m\hat{\Phi}^2}{2}+\int r\hat{\psi}d\hat{O}+\frac{\hat{O}^2}{2C}, \tag{2.273}$$

$$\hat{E}_p=\int\hat{P}_s d\hat{\Phi}=\frac{m\hat{\psi}^2}{2}+\int r\hat{\psi}d\hat{\Phi}+\frac{\hat{\Phi}^2}{2C}, \tag{2.274}$$

$$\hat{E}_f=\int\hat{F}_s d\hat{\psi}=\frac{m\hat{\upsilon}^2}{2}+\int r\hat{\upsilon}d\hat{\psi}+\frac{\hat{\psi}^2}{2C}, \tag{2.275}$$

$$\hat{E}_d = \int \hat{D}_s d\hat{\upsilon} = \frac{m\hat{w}^2}{2} + \int r\hat{w}d\hat{\upsilon} + \frac{\hat{\upsilon}^2}{2C}. \tag{2.276}$$

If the system is closed over the kinetic channel, i.e. $r = 0$, then energies

$$\hat{E}_s = \frac{m\hat{\Phi}^2}{2} + \frac{\hat{O}^2}{2C}, \quad \hat{E}_p = \frac{m\hat{\psi}^2}{2} + \frac{\hat{\Phi}^2}{2C}, \tag{2.277}$$

$$\hat{E}_f = \frac{m\hat{\upsilon}^2}{2} + \frac{\hat{\psi}^2}{2C}, \quad \hat{E}_d = \frac{m\hat{w}^2}{2} + \frac{\hat{\upsilon}^2}{2C}, \tag{2.278}$$

are conserved. If the system is open, motion-rest is also conserved but within the common bounds of the system and environment.