7. Kinematics-Dynamics of Non-uniform Circular Motion-Rest

In a general case of non-uniform revolution of the system of material points, potential-kinetic projections of motion of the points along the axes x and y has the form

$$\hat{\psi}_x = \hat{\psi}_{xp} + \hat{\psi}_{xk} = r\cos\varphi - ir\sin\varphi = re^{-i\varphi}, \qquad (2.166)$$

$$\hat{\psi}_y = \hat{\psi}_{yp} + \hat{\psi}_{yk} = r\sin\varphi + ir\cos\varphi = ire^{-i\varphi}, \qquad (2.166a)$$

where r is the distance to the axis of revolution, *cp* is the angular displacement.

The potential-kinetic velocity of motion of the points

$$\hat{\upsilon} = \hat{\upsilon}_p + \hat{\upsilon}_k = (-\omega ri)\mathbf{n} + \omega r\boldsymbol{\tau} \qquad (2.167)$$

has a structure of uniform motion but acceleration here acquires an additional term, reflecting the non-uniform aspect of the motion:

$$\hat{\mathbf{w}} = -\omega^2(r\mathbf{n} + ir\boldsymbol{\tau}) - i\varepsilon(r\mathbf{n} + ir\boldsymbol{\tau}) = -\omega^2\hat{\mathbf{r}} - i\varepsilon\hat{\mathbf{r}}, \qquad (2.168)$$

where $\hat{\mathbf{r}} = r\mathbf{n} + ir\boldsymbol{\tau}$ is a potential-kinetic radius.

The first term in the right-hand side is qualitative acceleration, acceleration of self-motion, a measure of uniform motion. The second term is quantitative acceleration, acceleration of non-self-motion, a measure of non-uniform motion. Thus, non-uniform circular motion is contradictory, being uniform-non-uniform. This statement is evidently valid for any non-uniform motion.

The terms in the expression for acceleration (2.168) will be rearranged as follows:

$$\hat{\mathbf{w}} = -(\omega^2 + i\varepsilon)r\mathbf{n} - (\omega^2 + i\varepsilon)ir\boldsymbol{\tau} = \hat{\mathbf{w}}_n + \hat{\mathbf{w}}_\tau, \qquad (2.169)$$

where

$$\hat{\mathbf{w}}_n = -(\omega^2 + i\varepsilon)r\mathbf{n} \qquad (2.169a)$$

is normal potential-kinetic acceleration;

$$\hat{\mathbf{w}}_\tau = (-i\omega^2 + \varepsilon)r\boldsymbol{\tau} \qquad (2.169b)$$

is tangential potential-kinetic acceleration.

In expression (2.169a), the first term $(-\omega^2)r\mathbf{n}$ is the centripetal potential acceleration, the second one $(-i\varepsilon)r\mathbf{n}$ is the normal kinetic acceleration (Fig. 2.8).

In tangential acceleration (2.169b), the first term $(-i\omega^2)r\boldsymbol{\tau}$ is the tangential kinetic acceleration, the second term $\varepsilon r\boldsymbol{\tau}$ is the tangential potential acceleration.

Now we will introduce the acceleration as the qualitatively-quantitative sum:

$$\hat{\mathbf{w}} = \hat{\mathbf{w}}_k + \hat{\mathbf{w}}_q, \qquad (2.170)$$

where

$$\hat{\mathbf{w}}_k = (-\omega^2)r\mathbf{n} + (-i\omega^2)r\boldsymbol{\tau}. \qquad (2.170a)$$

is a qualitative component of the acceleration;

$$\hat{\mathbf{w}}_q = \varepsilon r\boldsymbol{\tau} + (-i\varepsilon)r\mathbf{n} \qquad (2.170b)$$

is a quantitative component of its.

The qualitative acceleration (2.170a) is the potential-kinetic centripetal acceleration, whereas the quantitative acceleration (2.170b) is the potential-kinetic tangential acceleration.

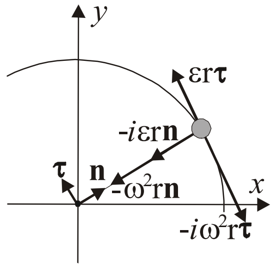

Fig. 2.8. Accelerations in non-uniform circular motion-rest or a vector graph of accelerations.

We will consider as well the potential-kinetic structure of the acceleration:

$$\hat{\mathbf{w}} = \mathbf{w}_p + \mathbf{w}_k, \tag{2.171}$$

where

$$\hat{\mathbf{w}}_p = -\omega^2 r\mathbf{n} + \varepsilon r\boldsymbol{\tau} \tag{2.171a}$$

is the potential acceleration;

$$\mathbf{w}_k = -i\varepsilon r\mathbf{n} - i\omega^2 r\boldsymbol{\tau} \tag{2.171b}$$

is the kinetic acceleration.

Specific acceleration has a similar structure. In particular, a normal-tangential or transverse specific acceleration has the form:

$$\hat{\boldsymbol{\omega}} = -(\omega^2 + i\varepsilon)\mathbf{n} + (-i\omega^2 + \varepsilon)\boldsymbol{\tau}\,. \tag{2.172}$$

Consequently, the longitudinal-transverse kinema of circular rest-motion is

$$\hat{\mathbf{F}} = m(-\omega^2 - i\varepsilon)r\mathbf{n} + m(-i\omega^2 + \varepsilon)r\boldsymbol{\tau}, \tag{2.173}$$

where

$$\hat{\mathbf{F}}_n = -m(\omega^2 + i\varepsilon)r\mathbf{n} \tag{2.173a}$$

is the normal or longitudinal potential-kinetic kinema;

$$\hat{\mathbf{F}}_\tau = m(-i\omega^2 + \varepsilon)r\boldsymbol{\tau} \tag{2.173b}$$

is the tangential or transverse potential-kinetic kinema (Fig. 2.9).

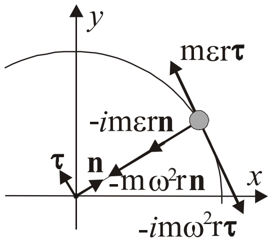

Fig. 2.9. A vector graph of kinemas in non-uniform

The kinema defines the longitudinal-transverse moment

$$\hat{\mathbf{M}} = J(-\omega^2 - i\varepsilon)\mathbf{n} + J(-i\omega^2 + \varepsilon)\boldsymbol{\tau}\,, \qquad (2.174)$$

where $J(-\omega^2)\mathbf{n}$ and $J(-i\varepsilon)\mathbf{n}$ are centripetal moments of rest-motion; $J(-i\omega^2)\boldsymbol{\tau}$ and $J\varepsilon\boldsymbol{\tau}$ are tangential moments of rest-notion.

The sum of moments $J(-\omega^2)\mathbf{n} + J(-i\omega^2)\boldsymbol{\tau}$ defines uniform rotation and $J(-i\varepsilon)\mathbf{n} + J\varepsilon\tau$ non-uniform rotation.

The axial moment in non-uniform rotation is defined by the equality

$$\hat{\mathbf{M}}_0 = J(\omega^2 + i\varepsilon)\mathbf{k}\,. \qquad (2.175)$$

In classical physics, the tangential potential moment $J\varepsilon\boldsymbol{\tau}$ is used only in the form of the axial moment with the norm $J\varepsilon\mathbf{k}$.

Kinematic non-uniform current is

$$\hat{\mathbf{I}} = \frac{\hat{\mathbf{F}}}{r} = m(-\omega^2 - i\varepsilon)\mathbf{n} + m(-i\omega^2 + \varepsilon)\boldsymbol{\tau} \qquad (2.176)$$

or

$$\hat{\mathbf{I}} = m\omega^2(\mathbf{n} + i\boldsymbol{\tau}) - mi\varepsilon(\mathbf{n} + i\boldsymbol{\tau})\,. \qquad (2.176a)$$

The laws of motion in circular motion are similar to the respective laws in rectilinear motion. In particular, the law of conservation of absolute-relative energy in the case of elastic-inelastic impact in rotational motion has the form

$$\frac{J_1\omega_1^2}{2} + \frac{J_2\omega_2^2}{2} + \frac{J_{12}(\omega_1 - \omega_2)^2}{2} = \frac{J_1'\omega_1'^2}{2} + \frac{J_2'\omega_2'^2}{2} + \frac{J_{12}'(\omega_1' - \omega_2')^2}{2}\,. \qquad (2.177)$$